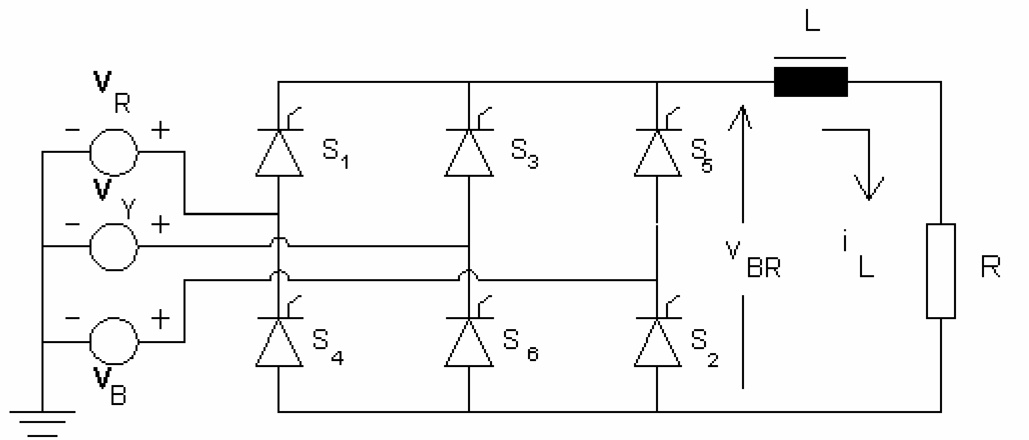
OPERATION OF A 3-PHASE FULLY-CONTROLLED RECTIFIER

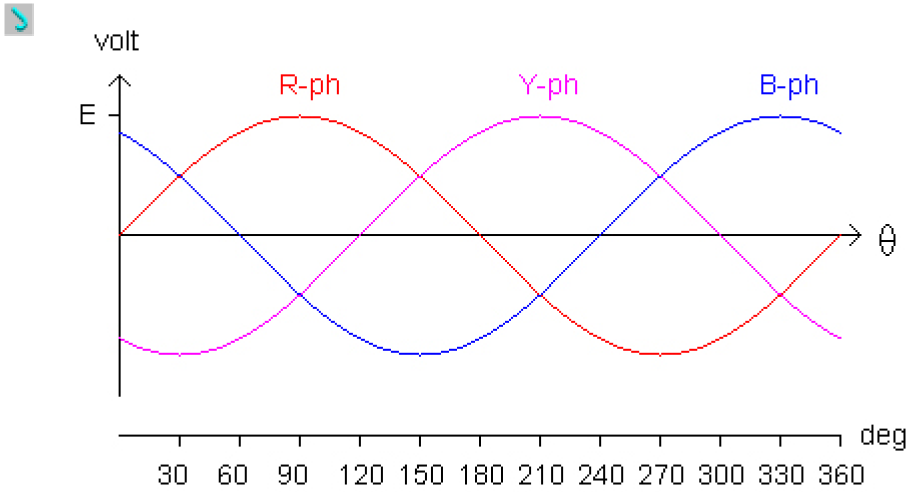
CIRCUIT OPERATION

The operation of a 3-phase fully-controlled bridge rectifier circuit is described in this page. A three-phase fully-controlled bridge rectifier can be constructed using six SCRs as shown below.



The three-phase bridge rectifier circuit has three-legs, each phase connected to one of the three phase voltages. Alternatively, it can be seen that the bridge circuit has two halves, the positive half consisting of the SCRs S_1, S_3 and S_5 and the negative half consisting of the SCRs S_2, S_4 and S_6. At any time, one SCR from each half conducts when there is current flow. If the phase sequence of the source be RYB, the SCRs are triggered in the sequence S_1, S_2 , S_3 , S_4, S_5 , S_6 and S_1 and so on.

The operation of the circuit is first explained with the assumption that diodes are used in place of the SCRs. The three-phase voltages vary as shown below.



Three-phase Voltages

Let the three-phase voltages be defined as shown below.

$$v_R(\theta) = E * Sin(\theta), \quad v_Y(\theta) = E * Sin(\theta - 120^{\circ}), \; and \; v_B(\theta) = E * Sin(\theta + 120^{\circ}).$$

It can be seen that the R-phase voltage is the highest of the three-phase voltages when q is in the range from 30^o to 150^o. It can also be seen that Y-phase voltage is the highest of the three-phase voltages when q is in the range from 150^o to 270^o and that B-phase voltage is the highest of the three-phase voltages when q is in the range from 270^o to 390^o or 30^o in the next cycle. We also find that R-phase voltage is the lowest of the three-phase voltages when q is in the range from 210^o to 330^o. It can also be seen that Y-phase voltage is the lowest of the three-phase voltages when q is in the range from 330^o to 450^o or 90^o in the next cycle, and that B-phase voltage is the lowest when q is in the range from 90^o to 210^o. If diodes are used, diode D_1 in place of S_1 would conduct from 30^o to 150^o, diode D_3 would conduct from 150^o to 270^o and diode D_5 from 270^o to 390^o or 30^o in the next cycle. In the same way, diode D_4 would conduct from 210^o to 330^o, diode D_6 from 330^o to 450^o or 90^o in the next cycle, and diode D_2 would conduct from 90^o to 210^o. The positive rail of output voltage of the bridge is connected to the topmost segments of the envelope of three-phase voltages and the negative rail of the output voltage to the lowest segments of the envelope.

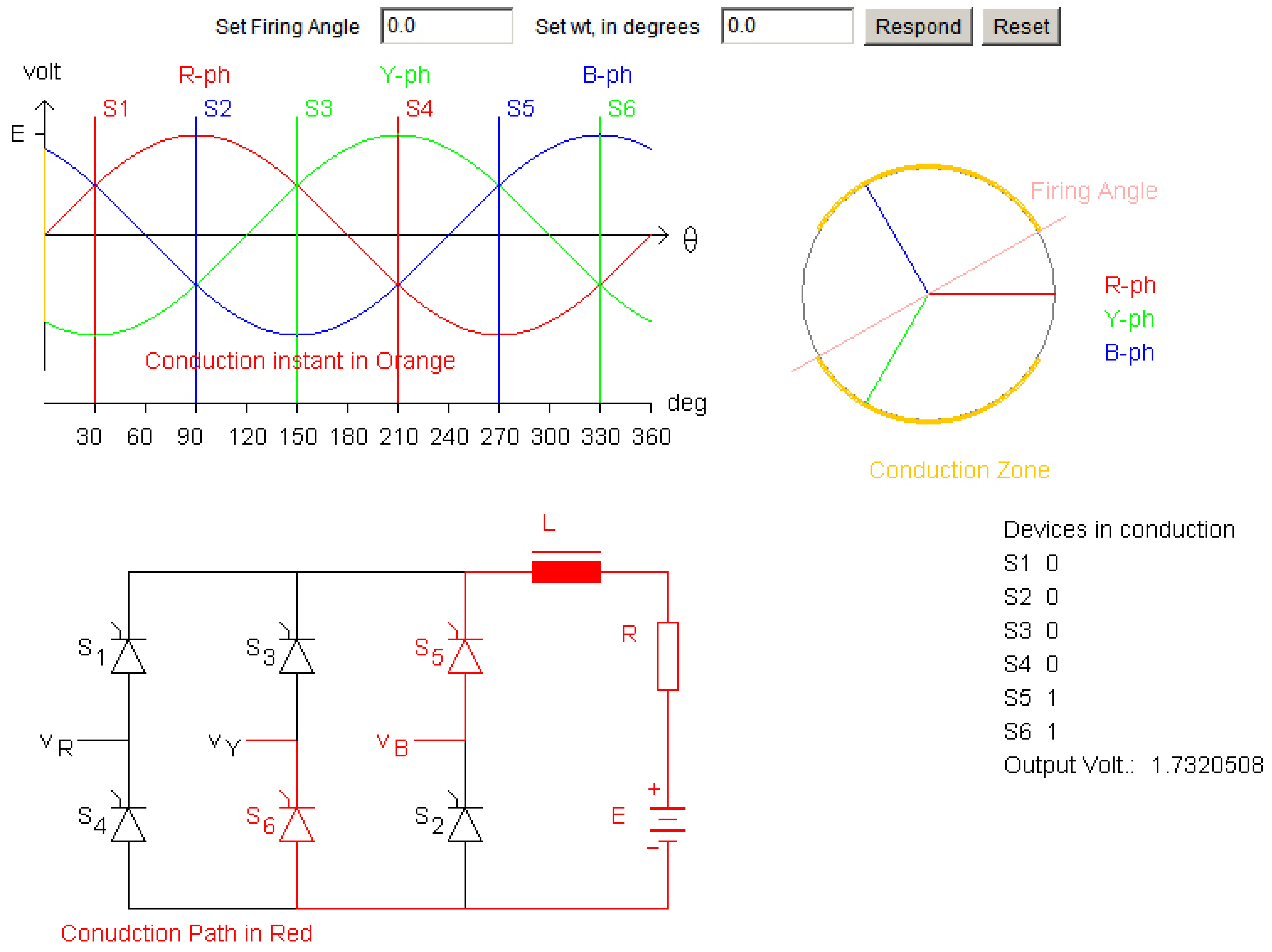
At any instant barring the change-over periods when current flow gets transferred from diode to another, only one of the following pairs conducts at any time.

Period, range of q	Diode Pair in conduction
30^o to 90^o	D_1 and D_6
90^o to 150^o	D_1 and D_2
150^o to 210^o	D_2 and D_3
210^o to 270^o	D_3 and D_4
270^o to 330^o	D_4 and D_5
330^o to 360^o and 0^o to 30^o	D_5 and D_6

If SCRs are used, their conduction can be delayed by choosing the desired firing angle. When the SCRs are fired at 0^o firing angle, the output of the bridge rectifier would be the same as that of the circuit with diodes. For instance, it is seen that D_1 starts conducting only after $q = 30^o$. In fact, it can start conducting only after $q = 30^o$, since it is reverse-biased before $q = 30^o$. The bias across D_1 becomes zero when $q = 30^o$ and diode D_1 starts getting forward-biased only after $q = 30^o$. When $v_R(q) = E*Sin(q)$, diode D_1 is reverse-biased before $q = 30^o$ and it is forward-biased when $q > 30^o$. When firing angle to SCRs is zero degree, S_1 is triggered when $q = 30^o$. This means that if a synchronizing signal is needed for triggering S_1, that signal voltage would lag $v_R(q)$ by 30^o and if the firing angle is a, SCR S_1 is triggered when $q = a + 30^o$. Given that the conduction is continuous, the following table presents the SCR pair in conduction at any instant.

Period, range of q	SCR Pair in conduction
$a + 30^o$ to $a + 90^o$	S_1 and S_6
$a + 90^o$ to $a + 150^o$	S_1 and S_2
$a + 150^o$ to $a + 210^o$	S_2 and S_3
$a + 210^o$ to $a + 270^o$	S_3 and S_4
$a + 270^o$ to $a + 330^o$	S_4 and S_5
$a + 330^o$ to $a + 360^o$ and $a + 0^o$ to $a + 30^o$	S_5 and S_6

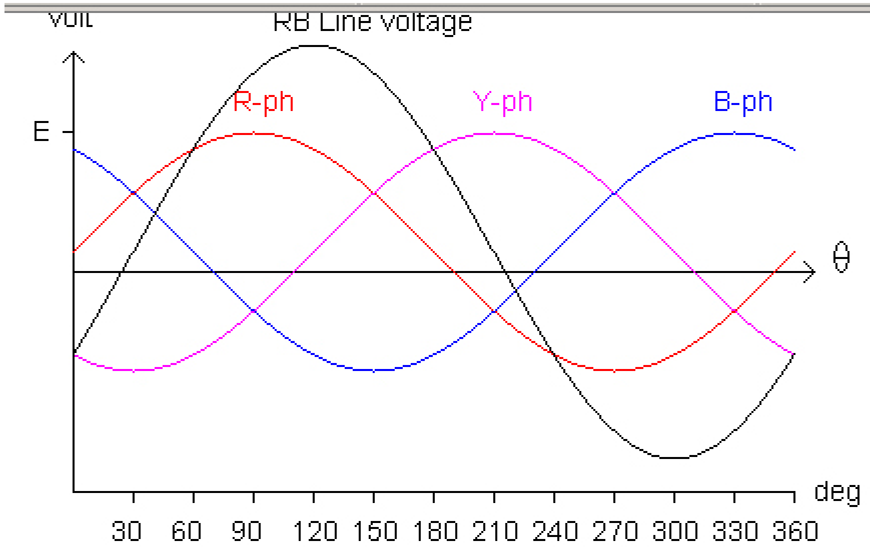
The operation of the bridge-rectifier is illustrated with the help of an applet that follows this paragraph. You can set the firing angle in the range 0° < firing angle < 180° and you can set the instantaneous angle also. The applet displays the SCR pair in conduction at the chosen instant. The current flow path is shown in red colour in the circuit diagram. The instantaneous angle can be either set in its text-field or varied by dragging the scroll-bar button. The rotating phasor diagram is quite useful to illustrate how the circuit operates. Once the firing angle is set, the phasor position for firing angle is fixed. Then as the instantaneous angle changes, the pair that conducts is connected to the thick orange arcs. One way to visualize is to imagine two brushes which are 120° wide and the device in the phase connected to the brush conducts. The brush that has "Firing angle " written beside it acts as the brush connected to the positive rail and the other acts as if it is connected to the negative rail. This diagram illustrates how the rectifier circuit acts as a commutator and converts ac to dc. The output voltage is specified with the amplitude of phase voltage being assigned unity value.



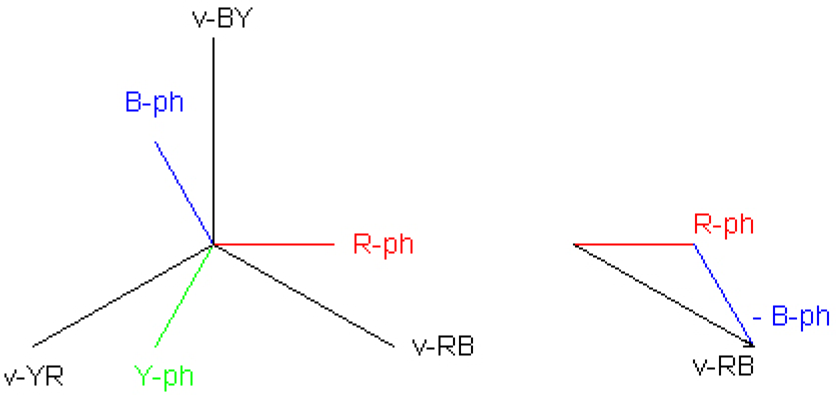
SYNCHRONIZING SIGNALS

To vary the output voltage, it is necessary to vary the firing angle. In order to vary the firing angle, one commonly used technique is to establish a synchronizing signal for each SCR. It has been seen that zero degree firing angle occurs 30^o degrees after the zero-crossing of the respective phase voltage. If the synchronizing signal is to be a sinusoidal signal, it should lag the respective phase by 30^o and then the circuitry needed to generate a firing signal can be similar to that described for single-phase. Instead of a single such circuit for a single phase rectifier, we would need three such circuits.

When the 3-phase source supply connected to the rectifier is star-connected, the line voltages and the phase voltages have a 30º phase angle difference between them, as shown below.



Three-phase Voltages



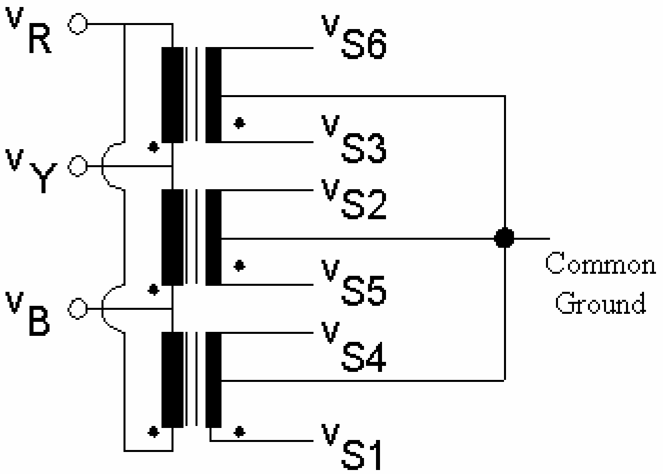
Phasor Addition

The line voltage can also be obtained as:

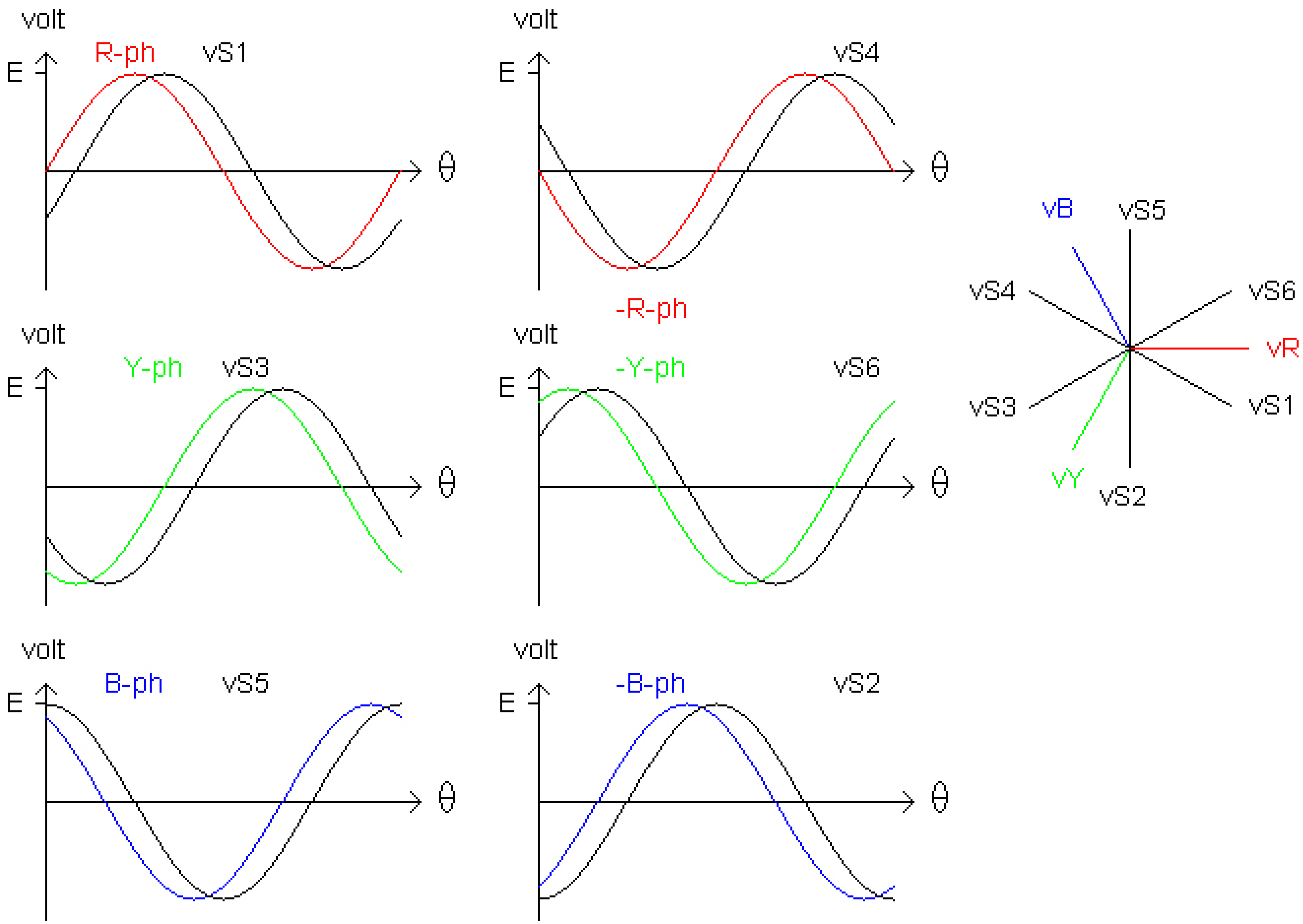
$$
\begin{aligned}
v_{RB}(\theta) &= v_R(\theta) - v_B(\theta) \\
&= E * Sin(\theta) - E * Sin(\theta + 120^\circ) \\
&= E * Sin(\theta) + \frac{E}{2} * Sin(\theta) - \frac{\sqrt{3}E}{2} * Cos(\theta) \\
&= \sqrt{3}E * Sin(\theta - 30^\circ).
\end{aligned}
$$

This line voltage lags the R-phase voltage by30^o and has an amplitude which is 1.732 times the amplitude of the phase voltage. The synchronizing signal for SCR S_1 can be obtained based on v_{RB} line voltage. The synchronizing signals for the other SCRs can be obtained in a similar manner.

To get the synchronizing signals, three control transformers can be used, with the primaries connected in delta and the secondaries in star, as shown below.



For S_1, voltage v_{S1} is used as the synchronizing signal. Voltage v_{S2} is used as the synchronizing signal for SCR S_2 and so on. The waveforms presented by the synchronizing signals are as shown below. The waveforms do not show the effect of turns ratio, since any instantaneous value has been normalized with respect to its peak value. For example, let the primary phase voltage be 240 V and then its peak value is 339.4 V. The primary voltage is normalized with respect to 339. V. If the peak voltage of each half of secondary is 10 V, the secondary voltage are normalized with respect to 10 V.



MATHEMATICAL ANALYSIS

Analysis of this three-phase controlled rectifier is in many ways similar to the analysis of single-phase bridge rectifier circuit. We are interested in output voltage and the source current. The average output voltage, the rms output voltage, the ripple content in output voltage, the total rms line current, the fundamental rms current, THD in line current, the displacement power factor and the apparent power factor are to be determined. In this section, the analysis is carried out assuming that the load current is a steady dc value.

AVERAGE OUTPUT VOLTAGE

Before getting an expression for the output voltage, it is preferable to find out how the output voltage waveform varies as the firing angle is varied. In one cycle of source voltage, six pairs conduct, each pair for 60^o. This means that the period for output waveform is one-sixth of the period of line voltage. The output waveform repeats itself six times in one cycle of input voltage. The waveform of output voltage can be determined by considering one pair. It is seen that when v_R(q) = E* Sin (q), SCR S1 and S6 conduct when q varies from 30^o + a to 90^o + a , where a is the firing angle. Then

$$\begin{aligned} v_o(\theta) &= v_R(\theta) - v_Y(\theta) \\ &= E * Sin(\theta) - E * Sin(\theta - 120^\circ) \\ &= \sqrt{3}E * Sin(\theta + 30^\circ), \ \ for\,(\alpha + 30^\circ) < \theta < (\alpha + 90^\circ). \end{aligned}$$

The waveform of output can be plotted for different firing angles. The applet below takes in the firing angle as an input and plots the output. The peak line-to-line voltage is marked as 'U' and the applet starts with the instant an SCR is fired and displays the output waveform for one input cycle period.

The average output voltage of the bridge circuit is calculated as follows, with a change in variable, where q = a + 60^o.

$$V_{o,avg}(\alpha) = \frac{\sqrt{3}E}{\pi/3}\int_{\alpha+\pi/3}^{\alpha+(2\pi)/3} Sin(\theta).d\theta = \frac{3}{\pi}U * Cos(\alpha).$$

In the expression above, U is the peak line-to-line voltage, whereas E is the amplitude of phase voltage of 3-phase supply.

RMS OUTPUT VOLTAGE

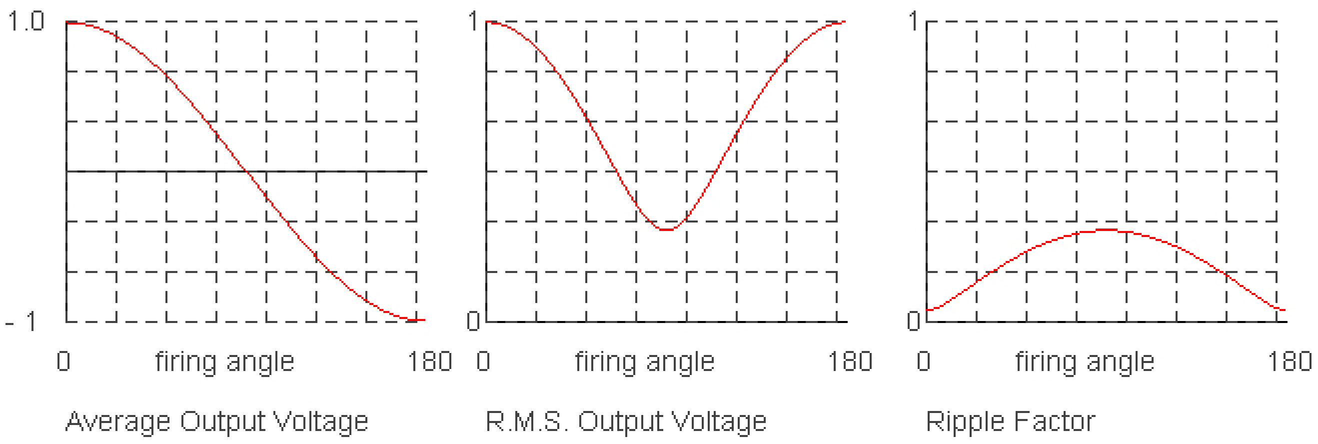
The rms output voltage is calculated as follows:

$$V_{o,rms}(\alpha) = U * \sqrt{\frac{3}{\pi}\int_0^{\pi/3} Sin^2(\theta + \frac{\pi}{3} + \alpha).d\alpha} \;= U * \sqrt{\frac{1}{2} + \frac{3\sqrt{3}}{4\pi} Cos(2\alpha)}.$$

The ripple factor of the output voltage is then:

$$RF(\alpha) = \frac{1}{V_{o,avg}(0^o)} * \sqrt{(V_{o,rms}(\alpha))^2 - (V_{o,avg}(\alpha))^2}$$

The applet below displays the average output voltage, the rms output voltage and the ripple factor for the case of continuous conduction through the load.



It is seen that the average output voltage is negative when firing angle exceeds 90^o. It means that power flow is from the dc side to the ac source. When the firing angle is kept in the region 0^o < a < 90^o, this circuit is said to be operating in the ***rectifier region***. When the firing angle is kept in the region 90^o < a < 180^o, this circuit is said to be operating in the ***inverter region***. When the circuit operates in the rectifier region, the net power flow is from the ac source to the dc link. In the inverter region, the net power flow is in the reverse direction. To operate in the inverter region, it is necessary to have a dc source present in the dc link which can provide the power that is fed back to the ac source.

RMS LINE CURRENT

The rms value of line current is relatively easy to find out if dc link current is ripple-free and steady. The load current is ripple-free if inductance in the dc link is relatively large. To maintain load current at any firing angle, it is assumed that the dc link contains a voltage source. Given that the resistance of the load circuit is zero, the voltage source should equal the average output voltage of the bridge circuit, if dc link current remains steady at some value. The waveforms shown below are based on the assumption that these conditions are met. It has been shown that if v_R(q) = E*Sin (q), SCR S_1 conducts when q varies from a + 30^o to a + 90^o and that SCR S_4 conducts when q varies from a + 210^o to a + 270^o. If the amplitude of dc load current is assigned to be unity, the line current waveform is then a rectangular pulse, remaining at + 1 from a + 30^o to a + 150^o, at - 1 from a + 210^o to a + 330^o, and zero elsewhere. The amplitude of fundamental in line current is then 3.464/p (which evaluates to nearly 0.78) and the amplitude of other odd harmonics is 3.464/np, where n is the odd harmonic number. When dc load current is steady and has a magnitude of unity, the rms line current is obtained as shown in equation (5). The rms value of the fundamental is obtained as shown in equation (6). Equation (6) is based on how trigonometric Fourier coefficients are defined for waveforms with quarter-wave symmetry. When the line current is a rectangular and symmetric, the phase current is the same as the line current and the fundamental component of phase current lags the phase voltage by an angle equal to the firing angle. Hence the displacement power factor is expressed as shown by equation (7). Since the line current is not sinusoidal, the apparent power factor, usually referred to just as the power factor in most of the texts, is less than DPF and is represented by equation (8). Since the line current is not sinusoidal, the distortion component in the line current has to be computed. This component, called the THD(Total Harmonic Distortion), is calculated as shown in equation (9).

$$I_{rms} = \sqrt{\frac{1}{\pi} * \int_{\pi/6}^{5\pi/6} d\theta} = 0.816. \quad (5)$$

$$I_{1,rms} = \frac{4}{\sqrt{2}\pi} * \int_{\pi/6}^{\pi/2} Sin(\theta).d\theta = 0.7797 \quad (6)$$

$$DPF = Cos(\alpha) \quad (7)$$

$$Apparent\ PF = \frac{I_{1,rms}}{I_{rms}} * DPF = 0.955 * Cos(\alpha). \quad (8)$$

$$THD = \sqrt{\frac{(I_{rms})^2}{(I_{1,rms})^2} - 1} = 0.311. \quad (9)$$